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- Since $f(2) = e^2 - 1$, we have $f^{-1}(e^2 - 1) = 2$.
- $f'(x) = e^x$, therefore $f'(2) = e^2$.
- The formula says that $(f^{-1})'(e^2 - 1) = \dfrac{1}{f'(f^{-1}(e^2 - 1))} = \dfrac{1}{f'(2)} = \dfrac{1}{e^2} = e^{-2}$

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- Multiplying both sides by x, we get $x + 4 = ex$ and $x - ex = -4$.
- Therefore $x(1-e) = -4$ and

$$x = \frac{-4}{1-e} = \frac{4}{e-1}.$$

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- Differentiating both sides with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(x^2+1)\ln(x^2+1) = 2x\ln(x^2+1) + \frac{2x(x^2+1)}{(x^2+1)} = 2x\Big[\ln(x^2+1)+1\Big].$$

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- Multiplying both sides by y, we get

$$\frac{dy}{dx} = y2x\left[\ln(x^2+1)+1\right] = (x^2+1)^{x^2+1}2x\left[\ln(x^2+1)+1\right]$$

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- $= \dfrac{2^x \ln 2 + 1}{2^x + x} + \dfrac{e^x}{\sqrt{1 - e^{2x}}}$

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- We use integration by parts with $u = x$, $dv = \cos x dx$. We get $du = dx$ and $v = \sin x$.
- Recall that $\int udv = uv - \int vdu$. Therefore
$$\int_0^{\pi/2} x\cos x dx = x\sin x\Big|_0^{\pi/2} - \int_0^{\pi/2} \sin x dx$$

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- $= \frac{\pi}{2} + \left[0 - 1\right] = \frac{\pi}{2} - 1.$

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- $\int \frac{x^2}{\sqrt{9-x^2}}dx = \int \frac{9\sin^2\theta}{3\cos\theta} 3\cos\theta d\theta = 9\int \sin^2\theta d\theta.$
- $= \frac{9}{2}\int (1-\cos(2\theta))d\theta = \frac{9}{2}\left[\theta - \frac{\sin(2\theta)}{2}\right] + C$

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- $\displaystyle = \frac{9}{2}\int (1-\cos(2\theta))d\theta = \frac{9}{2}\left[\theta - \frac{\sin(2\theta)}{2}\right] + C$
- We have $\theta = \sin^{-1}\frac{x}{3}$. Therefore $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}dx = \frac{9}{2}\left[\sin^{-1}\left(\frac{x}{3}\right) - \frac{2\sin\theta\cos\theta}{2}\right] + C$

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- Using a triangle, we get $\cos\theta = \frac{\sqrt{9-x^2}}{3}$ and $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}dx =$ $\displaystyle\frac{9}{2}\left[\sin^{-1}\left(\frac{x}{3}\right) - \frac{\frac{2}{9}\,x\sqrt{9-x^2}}{2}\right] + C = \frac{9}{2}\left[\sin^{-1}\left(\frac{x}{3}\right) - \frac{x\sqrt{9-x^2}}{9}\right] + C$

Question 8

8. *If you expand* $\dfrac{2x+1}{x^3+x}$ *as a partial fraction, which expression below would you get?*

a. $\dfrac{1}{x} + \dfrac{-x+2}{x^2+1}$

b. $\dfrac{2}{x} + \dfrac{1}{x^2+1}$

c. $\dfrac{-1}{x} + \dfrac{x}{x^2+1}$

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- $$\frac{2x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

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- $$\frac{2x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$
- Multiplying the above equation by $x(x^2+1)$, we get $2x+1 = A(x^2+1)+x(Bx+C) = Ax^2+A+Bx^2+Cx = (A+B)x^2+Cx+A$.

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- Comparing coefficients, we get $A = 1, \quad C = 2$, and $A+B = 0$. Therefore $B = -A = -1$.

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- $\dfrac{2x+1}{x(x^2+1)}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}$
- Multiplying the above equation by $x(x^2+1)$, we get $2x+1=A(x^2+1)+x(Bx+C)=Ax^2+A+Bx^2+Cx=(A+B)x^2+Cx+A$.
- Comparing coefficients, we get $A=1,\quad C=2$, and $A+B=0$. Therefore $B=-A=-1$.
- The partial fractions decomposition of $\dfrac{2x+1}{x(x^2+1)}$ is therefore $\dfrac{1}{x}+\dfrac{-x+2}{x^2+1}$.

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$$\int_0^2 \frac{1}{1-x} dx$$

is

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- We have $\lim_{t\to 1^+} \int_t^2 \frac{1}{1-x}dx = \lim_{t\to 1^+} [-\ln|1-x|\Big|_t^2$
  $= \lim_{t\to 1^+} [-\ln|-1| + \ln|1-t|] = -\infty$
- Therefore the integral $\int_0^2 \frac{1}{1-x}dx$ diverges.

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- $\Delta x = 1$.
- $M_4 = 1 \left[\frac{1}{4} + \frac{9}{4} + \frac{25}{4} + \frac{49}{4} \right] = \frac{84}{4} = 21$

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- We have initial amount $m_0 = 100$ and half life $t_{\frac{1}{2}} = 2$ days.
- The amount left after t days is given by $m(t) = m_0 e^{kt} = 100e^{kt}$ for some constant k.

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- To find the value of k, we use the fact that the half-life is 2 days. This tells us that $50 = 100e^{2k}$ or $\frac{1}{2} = e^{2k}$. Applying the natural logarithm to both sides, we get $\ln \frac{1}{2} = \ln e^{2k}$ or $-\ln 2 = 2k$.

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- To find the value of k, we use the fact that the half-life is 2 days. This tells us that $50 = 100e^{2k}$ or $\dfrac{1}{2} = e^{2k}$. Applying the natural logarithm to both sides, we get $\ln \frac{1}{2} = \ln e^{2k}$ or $-\ln 2 = 2k$.
- Therefore $k = \frac{-\ln 2}{2}$ and $m(t) = 100e^{-\frac{t \ln 2}{2}} = 100(e^{\ln 2})^{-\frac{t}{2}} = 100(2)^{-\frac{t}{2}}$

Question 11

11. *If 100 grams of radioactive material with a half–life of two days are present at day zero, how many grams are left at day three?*

- We have initial amount $m_0 = 100$ and half life $t_{\frac{1}{2}} = 2$ days.
- The amount left after t days is given by $m(t) = m_0 e^{kt} = 100e^{kt}$ for some constant k.
- To find the value of k, we use the fact that the half-life is 2 days. This tells us that $50 = 100e^{2k}$ or $\frac{1}{2} = e^{2k}$. Applying the natural logarithm to both sides, we get $\ln\frac{1}{2} = \ln e^{2k}$ or $-\ln 2 = 2k$.
- Therefore $k = \frac{-\ln 2}{2}$ and $m(t) = 100e^{-\frac{t\ln 2}{2}} = 100(e^{\ln 2})^{-\frac{t}{2}} = 100(2)^{-\frac{t}{2}}$
- After 3 days, we have $m(3) = 100(2)^{-\frac{3}{2}} = \dfrac{100}{3\sqrt{3}}$.

Question 12

12. *If* $x\frac{dy}{dx} + 3y = \frac{4}{x}$, *and* $y(1) = 10$, *find* $y(2)$.

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- The integrating factor is $e^{\int \frac{3}{x}dx} = e^{3\ln x} = x^3$.

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- Multiplying the standard equation by x^3, we get $x^3\frac{dy}{dx} + 3x^2y = 4x$ or $\frac{d(x^3y)}{dx} = 4x$.

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- Integrating both sides with respect to x, we get $x^3y = 4\frac{x^2}{2} + C = 2x^2 + C$.

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  $\frac{d(x^3y)}{dx} = 4x$.
- Integrating both sides with respect to x, we get
  $x^3y = 4\frac{x^2}{2} + C = 2x^2 + C$.
- Dividing across by x^3, we get $y = \frac{2}{x} + \frac{C}{x^3}$

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- Using the initial value condition $y(1) = 10$, we get $10 = y(1) = 2 + C$ or
  $C = 8$.

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- Using the initial value condition $y(1) = 10$, we get $10 = y(1) = 2 + C$ or $C = 8$.
- Therefore $y = \frac{2}{x} + \frac{8}{x^3}$ and $y(2) = 1 + 1 = 2$.

Question 13

13. *The solution to the initial value problem*

$$y' = x\cos^2 y \qquad\qquad y(2) = 0$$

satisfies the implicit equation

a) $\tan(y) = \frac{x^2}{2} - 2$ b) $\frac{ey}{2} = e^{\cos x} - e^{\cos 2}$ c) $\cos y = x - 1$
d) $\cos(y) = x + \cos(2)$ e) $e^{2y+1} = \arcsin(x - 2) + e$

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- We have $\int \sec^2 y dy = \int xdx$

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- We separate the variables $\frac{dy}{\cos^2 y} = xdx$
- We have $\int \sec^2 ydy = \int xdx$
- Therefore $\tan y = \frac{x^2}{2} + C$.
- Using the initial value condition, we get $y(2) = 0$ or $\tan 0 = \frac{2^2}{2} + C$, giving that $0 = 2 + C$ and $C = -2$.

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- We have $\int \sec^2 ydy = \int xdx$
- Therefore $\tan y = \frac{x^2}{2} + C$.
- Using the initial value condition, we get $y(2) = 0$ or $\tan 0 = \frac{2^2}{2} + C$, giving that $0 = 2 + C$ and $C = -2$.
- Therefore $\tan y = \frac{x^2}{2} - 2$.

Question 14

14. Use Euler's method with step size 0.1 to estimate $y(1.2)$ where $y(x)$ is the solution to the initial value problem

$$y' = xy + 1 \qquad y(1) = 0.$$

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- $x_0 = 1, \quad y_0 = 0$
- $x_1 = x_0 + h = 1.1, \quad y_1 = y_0 + h(x_0 y_0 + 1) = 0 + (0.1)(1 \cdot 0 + 1) = 0.1$

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- $x_2 = x_1 + h = 1.2, \quad y_2 = y_1 + h(x_1 y_1 + 1) = 0.1 + (0.1)((1.1)(0.1) + 1)$

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- $x_2 = x_1 + h = 1.2, \quad y_2 = y_1 + h(x_1 y_1 + 1) = 0.1 + (0.1)((1.1)(0.1) + 1)$
- $= 0.1 + 0.1(0.11 + 1) = 0.1 + 0.1(1.11) = 0.1 + 0.111 = 0.211$

Question 15

15. Find $\sum_{n=1}^{\infty} \frac{2^{2n}}{3 \cdot 5^{n-1}}$

$a)\ \frac{20}{3}$ $b)\ \frac{4}{15}$ $c)\ \frac{5}{4}$ $d)\ \frac{5}{3}$ $e)\ \frac{5}{12}$

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a) $\frac{20}{3}$ b) $\frac{4}{15}$ c) $\frac{5}{4}$ d) $\frac{5}{3}$ e) $\frac{5}{12}$

- $\sum_{n=1}^{\infty} \frac{2^{2n}}{3 \cdot 5^{n-1}} = \sum_{n=1}^{\infty} \frac{4^n}{3 \cdot 5^{n-1}} = \frac{4}{3} + \frac{4^2}{3 \cdot 5} + \dots$

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- This is a geometric series with $a =$ 1st term $= 4/3$ and $r =$ (2nd term)/ (1st term) $= 4/5$.

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- This is a geometric series with $a =$ 1st term $= 4/3$ and $r =$ (2nd term)/(1st term) $= 4/5$.
- Since $|r| < 1$, we have $\sum_{n=1}^{\infty} \frac{2^{2n}}{3 \cdot 5^{n-1}} = \frac{a}{1-r} = \frac{4/3}{1-4/5} = \frac{4/3}{1/5} = \frac{20}{3}$.

Question 16

16. Which of the following series converge conditionally?

$$\text{(I)} \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2} \qquad \text{(II)} \sum_{n=2}^{\infty} \frac{(-1)^n\, n}{\ln n} \qquad \text{(III)} \sum_{n=0}^{\infty} \frac{(-1)^n}{n} \; ?$$

(III) converges conditionally, (I) and (II) do not converge conditionally
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- $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges absolutely since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

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- $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges absolutely since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.
- $\sum_{n=2}^{\infty} \frac{(-1)^n\, n}{\ln n}$ diverges by the divergence test, since

$$\lim_{n\to\infty} \frac{n}{\ln n} = \lim_{x\to\infty} \frac{x}{\ln x} = (l'Hop) \lim_{x\to\infty} \frac{1}{1/x} = \infty.$$

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- $\sum_{n=2}^{\infty} \frac{(-1)^n\, n}{\ln n}$ diverges by the divergence test, since
$$\lim_{n\to\infty} \frac{n}{\ln n} = \lim_{x\to\infty} \frac{x}{\ln x} = (l'Hop) \lim_{x\to\infty} \frac{1}{1/x} = \infty.$$
- $\sum_{n=0}^{\infty} \frac{(-1)^n}{n}$ converges by the alternating series test, however it does not converge absolutely since $\sum_{n=0}^{\infty} \frac{1}{n}$ diverges.

Question 17

17. Which series below absolutely converges?

a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$ *b*) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)}$ *c*) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^3}$ *d*) $\sum_{n=1}^{\infty} \frac{\sqrt{n^3}}{n^2+1}$ *e*) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\pi^n}{3^n}$

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- $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)}$ does not converge absolutely since $\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$ diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$, ($n > \ln(n+1)$ for $n > 1$.)

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- $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^3}$ does not converge absolutely since $\sum_{n=1}^{\infty} \frac{n!}{n^3}$ diverges by the ratio test. ($\lim_{n\to\infty} \frac{(n+1)!/(n+1)^3}{n!/n^3} = \lim_{n\to\infty}(n+1) \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^3 = \infty > 1.$)

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- $\sum_{n=1}^{\infty} \frac{\sqrt{n^3}}{n^2+1}$ does not converge by comparison with $\sum_{n=1}^{\infty} \frac{n^{3/2}}{n^2} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ (which diverges because it is a p-series with $p < 1$).

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- $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$ converges absolutely since $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges.
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- $\sum_{n=1}^{\infty} \frac{(-1)^{n} n!}{n^3}$ does not converge absolutely since $\sum_{n=1}^{\infty} \frac{n!}{n^3}$ diverges by the ratio test. ($\lim_{n\to\infty} \frac{(n+1)!/(n+1)^3}{n!/n^3} = \lim_{n\to\infty}(n+1) \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^3 = \infty > 1.$)
- $\sum_{n=1}^{\infty} \frac{\sqrt{n^3}}{n^2+1}$ does not converge by comparison with $\sum_{n=1}^{\infty} \frac{n^{3/2}}{n^2} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ (which diverges because it is a p-series with $p < 1$).
- $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\pi^n}{3^n}$ diverges since it is a geometric series with $|r| = \frac{\pi}{3} > 1$.

Question 18

18. The interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{\sqrt{n}}$$

is

a) $[-4,-2)$ b) $(-4,-2)$ c) $(-1,1)$ d) $(2,4)$ e) $[2,4]$

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- Therefore the interval of convergence is $[-4,-2)$.

Question 19

19. If $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{(2n+1)!}$, find the power series centered at 2 for the function $\int_2^x f(t)\,dt$.

a) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+1}}{(n+1)(2n+1)!}$
b) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+1}}{(n^2)(2n+1)!}$
c) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{2n+1}}{(n+1)(2n)!}$
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e) The given function can not be represented by a power series centered at 2.

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e) The given function can not be represented by a power series centered at 2.

- $\int_2^x f(t)\, dt$ is the unique antiderivative $F(x) = \int \sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{(2n+1)!} dx$ with $F(2) = 0$.

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- We have $F(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int (x-2)^n dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{(x-2)^{n+1}}{n+1} dx + C$.

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- The condition that $F(2) = 0$ gives that $0 = F(2) = 0 + C$. Hence $C = 0$.

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- The condition that $F(2) = 0$ gives that $0 = F(2) = 0 + C$. Hence $C = 0$.
- Therefore $\int_2^x f(t) dt = F(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{(x-2)^{n+1}}{n+1} dx$.

Question 20

20. Which series below is the MacLaurin series (Taylor series centered at 0) for

$$\frac{x^2}{1+x}$$

a) $\sum_{n=0}^{\infty}(-1)^n\, x^{n+2}$ b) $\sum_{n=0}^{\infty} x^{2n+2}$ c) $\sum_{n=0}^{\infty} \frac{x^{n+2}}{n+2}$ $\sum_{n=2}^{\infty} \frac{(-1)^n x^{2n-2}}{n!}$ e) $\sum_{n=0}^{\infty}(-1)^n x^{2n}$

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- Using substitution we get $\frac{1}{1+x} = \sum_{n=0}^{\infty}(-x)^n = \sum_{n=0}^{\infty}(-1)^n x^n$
- Multiplying by x^2, we get $\frac{x^2}{1+x} = \sum_{n=0}^{\infty}(-1)^n x^{n+2}$.

Question 21

21. $\lim_{x \to 0} \dfrac{\sin(x^3) - x^3}{x^9} =$

Hint: Without MacLaurin series this may be a long problem.

a) $-\frac{1}{6}$ b) ∞ c) 0 d) $\frac{9}{7}$ e) $\frac{7}{9}$

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- $\sin x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \dots$

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- Therefore $\sin(x^3) = x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!} + \dots$.

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- Therefore $\sin(x^3) = x^3 - \dfrac{x^9}{3!} + \dfrac{x^{15}}{5!} - \dfrac{x^{21}}{7!} + \dots.$
- Hence $\dfrac{\sin(x^3) - x^3}{x^9} = \dfrac{-\frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!} + \dots}{x^9} = -\dfrac{1}{6} + \dfrac{x^6}{5!} - \dfrac{x^{12}}{7!} + \dots.$

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- Hence $\dfrac{\sin(x^3) - x^3}{x^9} = \dfrac{-\frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!} + \dots}{x^9} = -\dfrac{1}{6} + \dfrac{x^6}{5!} - \dfrac{x^{12}}{7!} + \dots.$
- Therefore $\lim_{x\to 0} \dfrac{\sin(x^3) - x^3}{x^9} = \lim_{x\to 0} \left[-\dfrac{1}{6} + \dfrac{x^6}{5!} - \dfrac{x^{12}}{7!} + \dots\right] = -\dfrac{1}{6}.$

Question 22

22. Which series below represents $\dfrac{\sin x}{x}$?

a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$ b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$ d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}$ e) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \binom{1/2}{n} x^{2n}$

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22. Which series below represents $\dfrac{\sin x}{x}$?

a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$ b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$ d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}$ e) $\sum_{n=0}^{\infty} (-1)^n \binom{1/2}{n} x^{2n}$

- $$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots.$$

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- $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots.$
- Therefore $\frac{\sin x}{x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots.$

Question 23

23. Which line below is the tangent line to the parameterized curve

$$x = \cos t + 2\cos(2t), \qquad y = \sin t + 2\sin(2t)$$

when $t = \pi/2$?
a) $y = 4x + 9$ b) $y = -4x - 7$ c) $y = x + 3$ d) $y = -x + 3$ e) $y = 1$

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- $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$
- $= \dfrac{\cos t + 4\cos(2t)}{-\sin t - 4\sin(2t)}$.

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- $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$
- $= \dfrac{\cos t + 4\cos(2t)}{-\sin t - 4\sin(2t)}$.
- When $t = \pi/2$, we have $\frac{dy}{dx} = \frac{-4}{-1} = 4$.

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- $= \dfrac{\cos t + 4\cos(2t)}{-\sin t - 4\sin(2t)}$.
- When $t = \pi/2$, we have $\frac{dy}{dx} = \frac{-4}{-1} = 4$.
- Also, when $t = \pi/2$, the corresponding point on the curve is $(-2, 1)$.

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- $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$
- $= \dfrac{\cos t + 4\cos(2t)}{-\sin t - 4\sin(2t)}$.
- When $t = \pi/2$, we have $\frac{dy}{dx} = \frac{-4}{-1} = 4$.
- Also, when $t = \pi/2$, the corresponding point on the curve is $(-2, 1)$.
- Therefore, when $t = \pi/2$, the tangent line has equation $y - 1 = 4(x + 2)$ or $y = 4x + 9$.

Question 24

24. Which integral below gives the arclength of the curve $x = 1 - 2\cos t$, $y = \sin^2\left(t/2\right)$, $0 \leq t \leq \pi$?

a) $\int_0^{\pi} \sqrt{4\sin^2 t + \sin^2(t/2)\cos^2(t/2)}\ dt$

b) $\int_0^{\pi} \sqrt{1 - 2\cos(t) + \cos^2(t) + \sin^4(t/2)}\ dt$

c) $\int_0^{\pi} \sqrt{1 - 2\cos(t) + \cos^2(t) + \sin^2(t/2)\cos^2(t/2)}\ dt$

d) $\int_0^{\pi} \sqrt{4\sin^2 t + \sin^4(t/2)}\ dt$

e) $\int_0^{\pi} \sqrt{\sin^2(t/2) - 2\sin^2(t/2)\cos(t)}\ dt$

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c) $\int_0^{\pi} \sqrt{1 - 2\cos(t) + \cos^2(t) + \sin^2(t/2)\cos^2(t/2)}\; dt$

d) $\int_0^{\pi} \sqrt{4\sin^2 t + \sin^4(t/2)}\; dt$

e) $\int_0^{\pi} \sqrt{\sin^2(t/2) - 2\sin^2(t/2)\cos(t)}\; dt$

- $L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}dt$

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d) $\int_0^{\pi} \sqrt{4\sin^2 t + \sin^4(t/2)}\ dt$

e) $\int_0^{\pi} \sqrt{\sin^2(t/2) - 2\sin^2(t/2)\cos(t)}\ dt$

- $L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}dt$
- $x'(t) = 2\sin t$ and $y'(t) = \frac{2}{2}\sin(t/2)\cos(t/2)$.

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c) $\int_0^{\pi} \sqrt{1 - 2\cos(t) + \cos^2(t) + \sin^2(t/2)\cos^2(t/2)}\ dt$

d) $\int_0^{\pi} \sqrt{4\sin^2 t + \sin^4(t/2)}\ dt$

e) $\int_0^{\pi} \sqrt{\sin^2(t/2) - 2\sin^2(t/2)\cos(t)}\ dt$

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- $L = \int_0^{\pi} \sqrt{4\sin^2 t + \sin^2(t/2)\cos^2(t/2)}dt$

Question 25

25. The point $(2, \frac{11\pi}{3})$ in polar coordinates corresponds to which point below in Cartesian coordinates?

a) $(1, -\sqrt{3}\,)$ b) $(-\sqrt{3}, 1)$ c) $(-1, \sqrt{3}\,)$ d) $(\sqrt{3}, -1)$ e) Since $\dfrac{11\pi}{3} > 2\pi$, there is no such point.

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- $x = r\cos\theta = 2\cos(11\pi/3) = 2\cos(5\pi/3) = 1$

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- $x = r\cos\theta = 2\cos(11\pi/3) = 2\cos(5\pi/3) = 1$
- $y = r\sin\theta = 2\sin(11\pi/3) = 2\sin(11\pi/3) = 2(-\sqrt{3}/2) = -\sqrt{3}$

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- $x = r\cos\theta = 2\cos(11\pi/3) = 2\cos(5\pi/3) = 1$
- $y = r\sin\theta = 2\sin(11\pi/3) = 2\sin(11\pi/3) = 2(-\sqrt{3}/2) = -\sqrt{3}$
- Therefore the point in Cartesian coordinates is $(1, -\sqrt{3})$.

Question 26

26. Find the equation for the tangent line to the curve with polar equation: $r = 2 - 2\cos\theta$ at the point $\theta = \pi/2$.

a) $y = 2 - x$ b) $y = 2 - \pi + 2x$ c) $y = 2 + \frac{\pi}{2} - x$ d) $y = 2 + 2x$ e) $y = 0$

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- A parameterization of this curve is given by
  $x = r\cos\theta = (2 - 2\cos\theta)\cos\theta = 2\cos\theta - 2\cos^2\theta$.
  $y = r\sin\theta = (2 - 2\cos\theta)\sin\theta = 2\sin\theta - 2\cos\theta\sin\theta$

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  $x = r\cos\theta = (2 - 2\cos\theta)\cos\theta = 2\cos\theta - 2\cos^2\theta$.
  $y = r\sin\theta = (2 - 2\cos\theta)\sin\theta = 2\sin\theta - 2\cos\theta\sin\theta$
- The slope at any point on the curve is given by
  $$\frac{dy/d\theta}{dx/d\theta} = \frac{2\cos\theta - 2[-\sin^2\theta + \cos^2\theta]}{-2\sin\theta - 4\cos\theta\sin\theta} = \frac{2\cos\theta + 2\sin^2\theta - 2\cos^2\theta}{-2\sin\theta + 4\sin\theta\cos\theta}.$$

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- When $\theta = \pi/2$, we get $\frac{dy/d\theta}{dx/d\theta} = \frac{0+2-0}{-2} = -1$.

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- When $\theta = \pi/2$, we get $\frac{dy/d\theta}{dx/d\theta} = \frac{0+2-0}{-2} = -1$.
- When $\theta = \pi/2$, the corresponding point on the curve is given by $x = 0$ and $y = 2$.

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- When $\theta = \pi/2$, we get $\frac{dy/d\theta}{dx/d\theta} = \frac{0+2-0}{-2} = -1$.
- When $\theta = \pi/2$, the corresponding point on the curve is given by $x = 0$ and $y = 2$.
- Therefore the tangent is given by $y - 2 = -x$ or $y = 2 - x$.

Question 27

27. Find the length of the polar curve between $\theta = 0$ and $\theta = 2\pi$

$$r = e^{-\theta}.$$

a) $\sqrt{2}(1 - e^{-2\pi})$ b) $\frac{1}{4}(1 - e^{-4\pi})$ c) $2e^{-4\pi}$ d) $2 - e^{-2\pi}$ e) $2\pi(1 + e^{-2\pi})$

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- The length of the polar curve is given by $L = \int_{\alpha}^{\beta} \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta$
- $\frac{dr}{d\theta} = -e^{-\theta}$, $\alpha = 0$, $\beta = 2\pi$.

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- The length of the polar curve is given by $L = \int_{\alpha}^{\beta} \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta$
- $\frac{dr}{d\theta} = -e^{-\theta}, \quad \alpha = 0, \qquad \beta = 2\pi.$
- $L = \int_0^{2\pi} \sqrt{e^{-2\theta} + e^{-2\theta}} d\theta = \int_0^{2\pi} e^{-\theta}\sqrt{2} d\theta = \sqrt{2}[-e^{-\theta}]_0^{2\pi} = \sqrt{2}[-e^{-2\pi} + e^0] = \sqrt{2}[1 - e^{-2\pi}].$